A nonlocal boundary value problem for semilinear leveling of mixed type

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ABSTRACT

This article presents the formulation of some mixed problems for an equation of the third-order composite type. Under certain conditions, the correctness of these problems in S. L. Sobolev spaces is proved for the coefficients and the right-hand side of the equation.

Keywords: mixed boundary value problems, equations of composite type, theorem, cylindrical domain, regular problems, differential equation, S.L. Sobolev spaces, Green's formula.

Keywords: gallery, local and total flow rates, internal and boundary conditions, sum of discontinuities of derived functions, one-sided integral Laplace transform, methods of the theory of functions of a complex variable.

Consider leveling:

Lu=K(x,y)Uyy+Uxx+α(x,y)Uy+b(x,y)U+m (1)$|u|^{p}U \ == f(x,y)$

Where K(x,y) is an infinitely differentiable function, and K(x,y) 0 at uK(x,y) at y, a(x,y)(D), b(x,y),m, p0.$\geq \geq 0, < 0 < 0 \epsilon \bar{C} \ \epsilon C^{1}(\bar{D} < 0 >$

Domain D– which consists at y0 of a straight square with vertices at points A (0; 0), B (1; 0), A 1 (0; 1>), B1 (1; 1) B, and at y0 is limited by the characteristics of leveling (1) <

S1=$\left\{(x,y): \frac{dx}{dy} = -\sqrt{-K}, y(0) = 0, y < 0\right\}$

S2= $\left\{(x,y): \frac{dy}{dx} = -\sqrt{-K}, y(0) = 0, y < 0\right\}$

Положим S = S1S∪ 2

Boundary problem. Find the solution of equation (1) in the domain D such that

U(0;y)=U(1;y)=0 (2)

$U(x;1) = p(x)U(x;y)/_{S}$ (3)

Everywhere = exp, uS. $\left[\frac{\lambda}{p+2}(-1 + y)\right], \lambda > 0 \in$

Where y0 through is denoted by the space of functions from the space , which satisfy the boundary conditions (2)-(3)< $W_2^1(D)W_2^1(D)$

Definition 1. The function and (x,y) is called the generalized solution of the problem $\in W_2^1(D)$ (1)-(3) if the integral value is fulfilled.

$$\int_D \left[-U_y(KV)_y - U_xV_x + a(x,y)U_yV + bUV + m|U|^3UV\right]dD = = \int_D fVdD$$

for any function V of . The existence of a generalized solution of the boundary value problem (1)-(3) established using the Galerkin method. Let $\{\overset{0}{W_2^1}(D)$U n(x,y)} be a set of functions from space having the property that all U$W_2^1(D)$n(x,y) are linearly independent , and their linear combinations are dense in this space . Such a multitude, as you know, is dry. [1], [4]

Let's take a look at the helper task

e=(x,y)= U$W_n e^{-\lambda y} W_{ny}$n(x,y) (5)

$W_n(x,y)/_{y\,=\,1} = \beta(x)W_n(x,y)/_{S}$ (6)

Solution of problem (5)-(6)

Wn(x,y)= $\int_S^y e^{\lambda\tau} U_n(x;\tau)d\tau + \frac{1}{\beta-1}\int_S \ e^{\lambda t} U_n(x,t)dt$

It is clear that Wn(x,y) are linearly independent. Indeed, if =0 for some set W 1,W2$\sum_{n=1}^{N} C_n W_n$,...... Wn then acting on this amount by the operator e, we have

$\sum_{n=1}^{N} C_n U_n(x,y) = 0, \Rightarrow$ Cn=0 , Vn

Clearly, Wn(x,y)is not difficult to get a ≤m markup ∈ $W_2^1(D)\|W_\mathrm{n}\|_{L_p(D)}^p \ \|U_\mathrm{n}\|_{L_p(D)}^p$

The crawl of Wn(x,y) satisfies conditions (6) for any n. The approximate solution of the problem (1)-(3) will be sought in the form

$U^N(x,y) = \sum_{n=1}^{N} C_n W_n(x,y)$

Where Cn are constants, which are defined from a system of nonlinear algebraic equations in the form

$(LU_1^N U_n)_0 = (f_1 U_n)_0$, n=1, N (7)

The solvability of this system of algebraic equations follows from the following estimates for approximate solutions and the lemma "of acute angle" [4]

Lemma 1. Let the conditions K(x,1)≥0 and inequality be met

2a(x,y)-K(x,y)-λK(x,y)≥0, V(x,y)D$\delta >\in$

Then the assessment is fair

$\|U^N\|_{W_2^1(D)}^2$+≤ (8)$\|U^N\|_{L_p(D)}^p K_2$

K_2it does not depend on n. Proof. Multiply (7) by Cn, summing up but n from 1 to N1, we get the identity

$\int_D \ e^{\lambda y}\, U_y^N L U^N$dD= dD (9)$\int_D \ e^{\lambda y}\, U_y^N f$

Integrating the left side of the equation (9) but the parts , we get

$\frac{1}{2}\int_D \ e^{\lambda y}$ dx-+ + $\Big[\lambda(U_y^N)^2 + (2a - \lambda K - Ky)(U_x^N)^2 + +\lambda(U^N)^2 + \frac{2m}{p}\|U^N\|^P\Big] dD - \frac{e^\lambda}{2}\int_0^1 (U_x^N)^2 dx +$

$+\frac{e^\lambda}{2}\int_0^1 K(x,1)\,(U_y^N)^2 \frac{e^\lambda}{2}\int_0^1 (U^N)^2\, dx \frac{1}{2}\int_S \ e^{\lambda y}\big[((U_y^N)^2 - K(U_y^N)^2 + m|U^N|^P +$

$+(U^N)^2)n_1 - 2(U_x^N)^2(U_y^N)^2 n_2\big]\, ds$

Where n=(n1:n2) is the unit vector of the internal normal to D. Using conditions (3) and lemma conditions, we obtain the inequality (8). Let us return to the question of the solvability of the system of equations (7). If we take it in the form =0 , where = (then, as we have just seen, multiplying, we get an estimate of ≥- $\partial\overrightarrow{F_m}\overrightarrow{(C)}\vec{C}\mathrm{C}_{1_m} \ldots . \mathrm{C}_{n_m})$ ($\vec{F}_m\vec{\mathrm{C}}_1\vec{C})_0$($\vec{F}_m\vec{\mathrm{C}}_1\vec{C})_0 \mathrm{K}_0 \|U\|_{W_2^1(D)}^2$

Due to the fact that the linear wrapper L() is a finite-dimensional space , there exists such that , $W_1, W_2, \ldots \ldots W_m \mathrm{K}_2(m)$

($\vec{F}_m\vec{\mathrm{C}}_1\vec{C})_0$ ≥ mean, the inequality of ≥ fulfilled

$\mathrm{K}_2(m)\sum_{S=1}^{N} C_S^2$ ($\vec{F}_m\vec{\mathrm{C}}_1\vec{C})_0 \mathrm{K}_2(m)\sum_{S=1}^{N} C_S^2 - \mathrm{K}_1 \geq 0$

If the large enough value A is the condition of the "acute angle", sufficient for the solvability of the system of levels (7).$\vec{\mathrm{C}}$

Then for any function f(x,y)L∈2(D) there is a generalized solution of problem (1)–(3).

Proof. By virtue of the estimate (8), the sequence is orgonic in the space L q $\{|U^N|^p U^N\}$ where +=1. Then, on the basis of (8) from the sequence, one can select the subsequence , converging in to some function (x,y) and the sequence weakly converges in L q to the function$\frac{1}{\mathrm{P}}\frac{1}{q}\{U^N\}W_2^1(D)U|U^N|^p U^N$ d(x,y)(D)

$\|U^N\|^p U^N$ (x,y) в L→ d^Nq(D)

The same theorem is that the veozhenie in L$W_2^1(D)$2(D) is continuous. Therefore, we can assume that the subsequence (x,y) is strong in U^NL2(D) and almost everywhere . Now we apply lemma 1.3 of the limit transition in a nonlinear glen in the case of , but where[2], [4]$|U^N|^p U^N$ q N = , q = it follows that q(x,y) = . Further, moving to the limit at N in (7) with n functioning, we have the equality $|U|^P U \ |U|^P U \rightarrow \infty$

$$\int_D \big[-U_y(KU_n)_y - U_x U_n + aUU_n + b(x,y)UU_n + m|U|^P UU_n\big]\, dD = \int_D fU_n dD$$

Where the function belongs to . Hence in view of the density in space it follows that the integral identity () is valid for any V(x,y)(D) theorem is proved.

$U(x,y)W_2^1(D)\{U_n\}W_2^1(D)U \in \overset{0}{W_2^1}$

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