A model of self-oscillation processes of a physical linear viscoelastic rod

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ABSTRACT

This article considers the problem of modeling self-oscillating processes of a physical linear viscoelastic rod in a gas flow, taking into account linear dependencies.

Keywords: viscoelasticity, rod, self-oscillations, physical linearity, aerodynamic linearity, Bubnov-Galerkin method, relaxation kernel, numerical method, critical velocity.

Kirish. The genetic theory of viscoelasticity created a wide opportunity to describe the dynamic processes of deformation of various materials. Due to the fact that struts are used as constructive elements in many fields of industry and technology, it is important to study their dynamic movements in various forms and to study designs for self-vibration and dynamic stability, taking into account the physical properties of the material.Setting the issue. Considering the property of physical linearity, we consider the issue of the self-oscillation process for a viscoelastic sturgeon [5]

$$\sigma = m_1(1-R^*)\varepsilon, \quad \varepsilon = u_x, \; u = -zw_x \qquad (1)$$

Or

$$\sigma = -m_1(1-R^*)zw_x \qquad (2)$$

where m_1 is the elasticity constant.

Taking into account the effect of aerodynamic linearity, the imposed aerodynamic load takes the following form [1]:

$$q = \frac{\chi p_\infty}{c_\infty}\left[V\frac{\partial w}{\partial x} + \frac{\partial w}{\partial t}\right]$$

Here $q = p - p_\infty, \quad k = \dfrac{\chi p_\infty}{c_\infty}$

$$q = k\left[V\frac{\partial w}{\partial x} + \frac{\partial w}{\partial t}\right] \qquad (3)$$

We build a model of self-oscillation processes in viscoelastic thin-walled structures.

In this case, we accept the hypothesis of flat sections for the bending moment and use the following formula [2]:

$$M_x = \int_{-h/2}^{h/2} b(x)\sigma_x z dz \qquad (4)$$

Substituting (2) into (4), we get:

$$M_x = -b(x)\left(1-R^*\right)\int_{-h/2}^{h/2} m_1 z w_{xx} z dz = -b(x)\left(1-R^*\right)m_1 w_{xx}\int_{-h\backslash 2}^{h/2} z^2 dz =$$

$$= -b(x)\left(1-R^*\right)m_1 w_{xx}\left.\frac{z^3}{3}\right|_{-h\backslash 2}^{h\backslash 2} = -b(x)\left(1-R^*\right)m_1 w_{xx}\frac{h^3}{12} =$$

$$= -\left(1-R^*\right)m_1\frac{b(x)h^3(x)}{12}w_{xx}$$

$$M_x = -E(1-R^*)J_2 w_{xx} \qquad (5)$$

b(x) is the width of the mast and h(x) is the height

$$J_2 = \frac{b(x)h^3(x)}{12}$$

Substituting (5) into the equilibrium equation [2] and passing to dimensionless coordinates, we obtain:

$$-\left(1-R*\right)\frac{\partial}{\partial x^2}\left[\frac{m_1 J_2^0 h_0}{a^2}g(x)w_{xx}\right] = m_0 F(x)\frac{h_0}{t_1^2}w_{tt} + kV\frac{h_0}{a}w_x + kz\frac{h_0}{t_1}w_t$$

$$-m_1 J_2^0\frac{h_0}{a^4}\left(1-R*\right)\frac{\partial}{\partial x^2}\left[g(x)w_{xx}\right] = m_0 F(x)\frac{h_0}{t_1^2}w_{tt} + kV\frac{h_0}{a}w_x + kz\frac{h_0}{t_1}w_t$$

$$\left(1-R*\right)\frac{\partial}{\partial x^2}\left[g(x)w_{xx}\right] + F(x)w_{tt} + Pw_x + \gamma w_t = 0 \qquad (6)$$

Here $w = h_0\overline{w},\ x = a\overline{x},\ t = t_1\bar{t},\ m(x) = m_0\overline{F(x)},\ h(x) = h_0\overline{h(x)},\ b(x) = b_0\overline{b(x)},$

$$J_2 = J_2^0 g(x), \quad g(x) = b(x)h^3(x), \quad J_2^0 = \frac{b_0 h_0^3}{12},$$

$$P = \frac{kVa^3}{m_1 J_2^{(0)}}, \quad t_1 = \sqrt{\left(m_0 a^4\right)/\left(m_1 J_2^{(0)}\right)}, \quad \gamma = \frac{kza^4}{m_1 J_2^{(0)} t_1},\ F(x) = b(x)h(x)$$

b(x)=c-a$_1$*x; h(x)=1-a*$_2$*x; c=5*

h_0 is the height value at the ends of the boom, b_0 is the width at the ends of the boom, m_0 is the mass value corresponding to the unit variable part of the boom.

Eigen-derivative linear IDTs (6), along with boundary [4] and initial conditions, represent a mathematical model of the auto-oscillating process problem for linear viscoelastic systems. It is required to find the critical speed Pkr, which leads to an increasing amplitude of oscillations.

We find the approximate solution by the Bubnova-Galerkin method. (6) We get the IDT solution in the following form

$$w = \sum_{k=1}^{N} u_k(t)\varphi_k(x) \qquad (7)$$

where $\varphi_к(x)$ are basic functions that satisfy given boundary conditions, $u_к(t)$ are unknown functions that need to be determined and depend on time.

(7) is put into (6) to find the unknown functions $u_к(t)$.

$$\left(1-R^*\right)\sum_{k=1}^{N}u_k(t)\left[g(x)\varphi_k''(x)\right]''+F(x)\sum_{k=1}^{N}\ddot{u}_k(t)\varphi_k(x)+P\sum_{k=1}^{N}u_k(t)\varphi_k'(x)+\gamma\sum_{k=1}^{N}\dot{u}_k(t)\varphi_k(x)=0$$

Multiplying by $\varphi_i(x)$ and integrating over x, we get:

$$\left(1-R^*\right)\sum_{k=1}^{N}u_k(t)\int_0^1\left[g(x)\varphi_k''(x)\right]''\varphi_i(x)dx+\sum_{k=1}^{N}\ddot{u}_k(t)\int_0^1 F(x)\varphi_k(x)\varphi_i(x)dx+$$

$$+P\sum_{k=1}^{N}u_k(t)\int_0^1\varphi_k'(x)\varphi_i(x)dx+\gamma\sum_{k=1}^{N}\dot{u}_k(t)\int_0^1\varphi_k(x)\varphi_i(x)dx=0$$

Introducing notations for integrals, we arrive at the following linear system of simple IDTs

$$\sum_{k=1}^{N}\left[a_{ki}\ddot{u}_k(t)+\gamma b_{ki}\dot{u}_k(t)+\omega_{ki}\left(1-R^*\right)u_k(t)+Pd_{ki}u_k(t)\right]=0,\ i=\overline{1,N} \qquad (8)$$

Here $a_{ki}=\int_0^1 F(x)\varphi_k(x)\varphi_i(x)dx,\qquad b_{ki}=\int_0^1\varphi_k(x)\varphi_i(x)dx,$

$$\omega_{ki}=\int_0^1\left[d(x)\varphi_k''(x)\right]''\varphi_i(x)dx,\qquad d_{ki}=\int_0^1\varphi_k'(x)\varphi_i(x)dx,$$

Integrating the linear system in the Rjanitsyna–Koltunov kernel (8) into analytical substitutions, taking into account the variation of the physical-mechanical parameters of the structure in a wide range $R(t)=A\cdot e^{-\beta t}t^{\alpha-1}$, $A>0$, $\beta>0$, $0<\alpha<1$ based on the numerical method [3].

Summary. The analysis of physical linear problems shows that the value of the critical speed is fully dependent on the elastic and visco-elastic states of the structure.
A general calculation algorithm was developed and implemented on a computer.

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