Using T- transformation for Solving System of PDEs with Fractional order

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ABSTRACT

In this paper, we have dealt with a new technique for solving a system of linear and non-linear PDEs with fractional orders. This method is a combination of the T- transformation. Where the T-transformation is considered generalizations of the previous integral transformations. The method and the iterative method are then called the iterative T-transformation. This is a method that gives appropriate solutions and free from rounding errors and this is done by comparing it with other methods. They are useful in reducing digital accounts and we reinforce this with some examples.

Keywords: Iterative method ,T- transform method, mittage-Leffler function Fractional non-linear partial differential equations,

1. Introduction

In recent years, There are many problems in math physics and engineering such as the physics of polymers have been successfully analyzed by partial differential equations (PDEs)[1].But when these problems interfere with differential equations with non-integer order (FDEs), things will be more eye-catching. To solve FDEs, new and effective methods must be found. Also recently Jafari, Daftardar-Gejjii introduced a new iterative method [2]. This method solves PDEs for integers and fractional order. In this work, we have considered a new method called iterative g-transformation method (ITTM). Where this technique consolidates two methods, the g-transformation and the iterative method, it is worth noting that (ITTM) It is applied easily and without assumptions. Reversing the method of separating variables that include initial conditions and limits. It is possible to legitimize the outcomes obtained by the proposed method using boundary conditions. The outcomes so far are exceptionally reassuring and dependable in light of the fact that it works effectively. To solve systems of partial differential equations with nonlinear fractional orders we used lLGM. And we gave some examples to verify the work and performance of this method. Then, at this point, the outcomes are compared with those obtained through previous techniques.

2. Important definitions:

2.1. Definition . The function $f(x)$, $x > 0$ it is real in the space C_β, $\beta \in R$ **,** if there exists a real number $m > \beta$, such that $f(x) = x^m f_1(t)$ where $f_1 \in C[0, \infty]$. Clearly $C_\beta \subset C_\mu$ if $\beta \leq \mu$

2.2. Definition . The function $f(x)$, $x > 0$ is real in the space C_β^j $j \in N \cup \{0\}$ if it is $f^{(j)} \in C_\beta$.

2.3. Definition [**2**]. let f be a function such that $f \in C_{-1}^k$,the left sided fractional integral of Riemann–Liouville of order $\eta \geq 0$, $f \in C_\eta$ is defined in the following form:

$$I^{\eta}f(y)=\begin{cases}\dfrac{1}{\Gamma(\eta)}\displaystyle\int_0^y \frac{f(\omega)}{(y-\omega)^{1-m}}dy & ,\eta>0\,,\ \ y>0\\ f(y)\quad , & \eta=0\end{cases}\qquad (1)$$

2.4. Definition[3] let f be a function such that $f\in C_{-1}^{k}$, $k\in N\cup\{0\}$,then we can defined the left sided fractional Caputo derivative of f, in the following form,

$$D^{\eta}f(y)=\frac{\partial^{\eta}f(y)}{\partial y^{\eta}}=\begin{cases}I^{k-\eta}\left[\dfrac{\partial^{k}f(y)}{\partial y^{k}}\right], & k-1<\eta<k\,,k\in N\\ \dfrac{\partial^{k}f(y)}{\partial y^{k}} & ,\quad \eta=k\end{cases}\qquad (2)$$

1) $I_y^{\eta}f(t,y)=\frac{1}{\Gamma(\eta)}\int_0^y\frac{f(t,y)}{(y-s)^{1-\eta}}$, $\eta>0$, $y>0$

2) $D_y^{\eta}f(t,y)=I_y^{k-\eta}\frac{\partial^k f(t,y)}{\partial y^k}$, $k-1<\eta<k$

2.5. Definition: [5] The Mittag-Leffler function $E_{\eta}(z)$ when $\eta>0$, we can define it with the following string representation , So that this function takes the entire complex level

$$E_{\eta}(z)=\sum_{0}^{\infty}\frac{z^{i}}{\Gamma(\eta i+1)}$$

2.6. Definition: [4]

Let f(t) be a continuous function and $t\in[\,0\,,\infty\,)$ we can defined the general T- transformation $T(f(t),p,q)$ for a piecewise function f(t) by the following integral

$$T\big(f(t,q)\big)=p(s)\int_0^{\infty}e^{-q(s)t}\ f(t)dt\quad ,\qquad p(s)\neq 0\qquad (3)$$

such that the integral is convergent for some q(s), s is positive constant, and

$$\|T(f(t))\|\le\frac{p(s)M}{L-q(s)}\quad ,\qquad q(s)\neq L$$

where $\|.\|$ is anorm on dual of R and it is defined as

$$\big\|\big(f(t)\big)\big\|=\max|f(t)|\qquad ,t\in[0,\infty)$$

2.7 Definition : The T-transformation of T[f (t)] of a given Riemann–Liouville partial integral is defined in the following form:

$$T\{I^{\eta}f(t)\}=\big(q(s)\big)^{-\eta}F(p,q)\qquad (4)$$

2.8. Definition :

let f be a function .The T- transform T(f (y)) of the Caputo fractional derivative is defined in the following form

$$T(D^{\eta}f(t)=q^{\eta}F(p,q)-p(s)\sum_{r=0}^{n-1}q^{(\eta-r-1)}f^{(r)}(0)\quad ,\quad n-1<\eta\le n\qquad (5)$$

3. Use iterative T-transformation to solve a system PDEs.

In this section we explain the importance of this method for solving a system PDEs with initial conditions

$$D_y^{\eta i}u_i(\bar{t},y)=V_i\big(u_1(\bar{t},y),\ldots\ldots,u_n(\bar{t},y)\big),\qquad k_i-1<\eta_i<k_i\qquad (6)$$

$$\frac{\partial^{\lambda i}u_i(t,0)}{\partial y^{\lambda i}}=h_{i\lambda i}\,,\qquad \lambda_i=0,1\ldots\ldots,k_i-1\,,k_i\ \in N\qquad (7)$$

We take the T-transform for both sides of the equation .we get

$T\big[D_y^{\eta i}u_i(\bar{t},y)\big]=T\big[V_i\big(u_1(\bar{t},y),\ldots\ldots,u_n(\bar{t},y)\big)\big]$, $i=0,1,\ldots\ldots,n$

by Definition 8 and the initial conditions (7) .we get.

$$q^{\alpha i}T[u_i(\bar{t},y)] - p(s)\sum_{r=0}^{mi-1} q^{\alpha i-r-1}u_i^{(r)}(\bar{t},0) = T[V_i u_1(\bar{t},y),\dots,u_n(\bar{t},y)] \;\; i = 1,2,\dots (8)$$

Taking the T- inverse on both sides of Eq. (8) we get

$$u_i(\bar{t},y) = T^{-1}\left[p(s)\sum_{r=0}^{mi-1} q^{-r-1}u_i^{(r)}(\bar{x},0)\right] + T^{-1}\left[q^{-\alpha i}T[V_i u_1(\bar{t},y),\dots,u_n(\bar{t},y)]\right]$$

$$= f_i + R_i\big(u_1(\bar{t}.y),\dots,u_1(\bar{t},y)\big)\;, \qquad i = 1,2,\dots,n \qquad (9)$$

$$u_i(\bar{t},y) = f_i + R_i\big(u_1(\bar{t}.y),\dots,u_1(\bar{t},y)\big)\;, \qquad i = 1,2,\dots,n \qquad (10)$$

Where $f_i = T^{-1}\left[p(s)\sum_{r=0}^{mi-1} q^{-r-1}u_i^{(r)}(\bar{t},0)\right]$, $\quad i = 1,2,\dots,n$

$$R_i\big(u_1(\bar{t},y),\dots,u_n(\bar{t},y)\big) = T^{-1}\left[q^{-\alpha i}g[V_i u_1(\bar{t},y),\dots,u_n(\bar{t},y)]\right]$$

$$u_i(\bar{t},y) = \sum_{j=0}^{\infty} u_{ij}(\bar{t},y) \quad , \qquad i = 1,2,\dots,n \qquad (11)$$

nonlinear operators R_i can be written in the following from.

$$R_i\left(\sum_{j=0}^{\infty} u_{1j}(t,y),\dots,\sum_{j=0}^{\infty} u_{nj}(\bar{t},y)\right)$$
$$= R_i\big(u_{10}(\bar{t},y),\dots,u_{n0}(\bar{t},y)\big) + \sum_{j=1}^{\infty}\{R_i\left(\sum_{k=0}^{j} u_{1k}(t,y),\dots,\sum_{k=0}^{j} u_{nk}(\bar{t},y)\right)$$
$$- R_i\left(\sum_{k=0}^{j-1} u_{1k}(t,y),\dots,\sum_{k=0}^{j-1} u_{nk}(\bar{t},y)\right) \qquad (12)$$

in view of Eqs. (11) and (12), Eq. (10) is equivalent to

$$\sum_{j=0}^{\infty} u_{ij}(\bar{t}) = f_i + R_i(u_{10}(\bar{t},y),\dots,u_{n0}(\bar{t},y)$$
$$+\sum_{j=1}^{\infty}\{R_i\left(\sum_{k=0}^{j} u_{1k}(t,y),\dots,\sum_{k=0}^{j} u_{nk}(\bar{t},y)\right) - R_i\left(\sum_{k=0}^{j-1} u_{1k}(t,y),\dots,\sum_{k=0}^{j-1} u_{nk}(\bar{t},y)\right)\} \qquad (13)$$

Then we define the frequency.

$$\begin{cases} u_{i0}(\bar{t},y) = T^{-1}\left[p(s)\sum_{k=0}^{m_i-1} q^{-r-1}u_i^{(r)}(\bar{t},y)\right] \qquad (14) \\ u_{i1}(\bar{t},y) = T^{-1}[q^{-\eta i}T\big[R_i(u_{10}(\bar{t},y),\dots,u_{n0}(\bar{t},y)\big]] \\ u_{i(m+1)}(\bar{t},y) = T^{-1}[q^{-\eta i}T\big[R_i(u_{10}(\bar{t},y) + \dots + u_{1m}(\bar{t},y),\dots,u_{n0}(\bar{t},y) + \cdots + u_{nm}(\bar{t},y)))\big]] \\ \quad -T^{-1}[q^{-\eta i}T\left[R_i\big(u_{10}(\bar{t},y) + \dots + u_{1(m-1)}(\bar{x},y),\dots,u_{n0}(t,y) + \cdots + u_{n(m-1)}(\bar{t},y))\big)\right]] \end{cases}$$

Then .

$$u_{i1}(\bar{t},y) + \cdots + u_{i(k+1)}(\bar{t},y) = T^{-1}[q^{-\eta i}T\big[R_i(u_{10}(\bar{t},y) + \cdots + u_{1k}(\bar{t},y),\dots,u_{n0}(\bar{x},y) + \cdots + u_{nk}(\bar{t},y)))\big]]$$

$$u_i(\bar{t},y) \cong u_{i1}(\bar{t},y) + \cdots + u_{i,n}(t,y) \quad , \quad i = 1,2,\dots,n$$

We notice that the solutions of the series above converge very quickly. Both al Jafari and Daftardar-Gejji have brought a classic approach to this type of chain

4. Examples :

In this part we study the possibility of applying the iterative transform in solving a system of differential equations with fractional orders.

4.1. Example . To solve the system of linear FPDEs [8]:

$$D_y^{\eta} u - \omega_t + \omega + u = 0$$
$$D_y^{\mu} z - u_t + \omega + u = 0 \qquad (0 < \eta, \mu \le 1) \qquad (15)$$

And initial conditions:

$u(t,0) = \sinh(t) \quad , \quad \omega(t,0)\omega = \cosh(t)$

when $\mu = \eta = 1$, then the exact solution is

$u(t,y) = \sinh(t-y) \quad , \quad \omega(t,y) = \cosh(t-y)$

$$u(t,y) = T^{-1}[p(s)q^{-1}u(t,0)] + T^{-1}\left[q^{-\eta}T[\omega_t(t,y) - \omega(t,y) - u(t,y)]\right]$$
$$\omega(t,y) = T^{-1}[p(s)q^{-1}\omega(t,0)] + T^{-1}\left[q^{-\mu}T[u_t(t,y) - \omega(t.y) - u(t,y)]\right]$$

With applied algorithm in Eq (14) we get.

$$\{u_0(t,y) = \sinh(t) \quad , \omega_0(t,y) = \mathrm{coh}(t), \quad \begin{cases} u_1(t,y) = -\dfrac{\cosh(t)y^{\eta}}{\Gamma(\eta+1)} \\ \omega_1(t,y) = -\dfrac{\sinh(t)y^{\mu}}{\Gamma(\mu+1)} \end{cases}$$

$$u_2(t,y) = -\frac{\cosh(t)\,y^{\eta+\mu}}{\Gamma(\eta+\mu+1)} + \frac{\sinh(t)\,y^{\eta+\mu}}{\Gamma(\eta+\mu+1)} + \frac{\cosh(t)\,y^{2\eta}}{\Gamma(2\eta+1)}$$

Then we get the solution sequentially. Use

$$u(t,y) = u_0(t,y) + u_1(t,y) + u_2(t,y) + \cdots + \sinh(t)\left(1 + \frac{y^{\eta+\mu}}{\Gamma(\eta+\mu+1)} + \cdots\right) - \cosh(t)\left(\frac{y^{\eta}}{\Gamma(\eta+1)} + \frac{y^{\eta+\mu}}{\Gamma(\eta+\mu+1)} - \frac{y^{2\eta}}{\Gamma(2\eta+1)} + \cdots\right) \qquad (16)$$

$$\omega(t,y) = \omega_0(t,y) + \omega_1(t,y) + \omega_2(t,y) + \cdots = \cosh(t)\left(1 + \frac{y^{\eta+\mu}}{\Gamma(\eta+\mu+1)} + \cdots\right) - \sinh(t)\left(\frac{y^{\eta}}{\Gamma(\eta+1)} + \frac{y^{\alpha+\mu}}{\Gamma(\eta+\mu+1)} - \frac{y^{2\eta}}{\Gamma(2\eta+1)} + \cdots\right) \qquad (17)$$

put $\eta = \mu$ in Eqs. (16) and (17), we reproduce the solution of [7] as follows:

$$u(t,y) = \sinh(t)\left(1 + \frac{y^{2\eta}}{\Gamma(2\eta+1)} + \cdots\right) - \cosh(t)\left(\frac{y^{\eta}}{\Gamma(\eta+1)} + \frac{y^{3\eta}}{\Gamma(3\eta+1)} + \cdots\right) \quad (18)$$
$$\omega(t,y) = \cosh(t)\left(1 + \frac{y^{2\eta}}{\Gamma(2\eta+1)} + \cdots\right) - \sinh(t)\left(\frac{y^{\eta}}{\Gamma(\eta+1)} + \frac{y^{3\eta}}{\Gamma(3\eta+1)} + \cdots\right) \quad (19)$$

Now put $\alpha = 1$ in Eq. (18) and (19), we get.

$$u(t,y) = \sinh(t)\left(1 + \frac{y^2}{2!} + \frac{y^4}{4!}\ldots\right) - \cosh(t)\left(y + \frac{y^3}{3!} + \frac{y^5}{5!} + \cdots\right) = \sinh(t-y)$$
$$\omega(t,y) = \cosh(t)\left(1 + \frac{y^2}{2!} + \frac{y^4}{4!}\ldots\right) - \sinh(t)\left(y + \frac{y^3}{3!} + \frac{y^5}{5!} + \cdots\right) = \cosh(t-y)$$

4.2. To solve the system of nonlinear FPDEs [7]:

$$D_z^{\eta} = \omega_t h_y - \omega_y h_t = -u$$
$$D_z^{\mu} v + u_t h_y + u_y h_t = \omega \qquad (20)$$
$$D_z^{\gamma} h + u_t \omega_y + u_y \omega_t = h \quad , \quad (0 < \eta, \mu, \gamma \le 1),$$

And initial conditions.

$$u(t,y,o) = e^{t+y} \quad , \quad \omega(t,y,0) = e^{t-y} \quad , \quad h(t,y,0) = e^{-t+y}. \qquad (21)$$

The exact solution, when $\eta = \mu = \gamma = 1$, is
$u(t, y, z) = e^{t+y-z}$, $\omega(t, y, z) = e^{t-y+z}$, $h(t, y, z) = e^{-t+y+z}$.
As in Example 1 above, we construct the following:

$$u(t,y,z) = T^{-1}[p(s)q^{-\alpha}u(t,y,0)] + T^{-1}\left[q^{-\alpha}T\left[-u(t,y,z) - \omega_t(t,y,z)h_y(t,y,z) + \omega_y(t,y,z)h_t(t,y,z)\right]\right]$$

$$\omega(t,y,z) = T^{-1}[p(s)q^{-\alpha}\omega(t,y,0)] + T^{-1}\left[q^{-\alpha}T\left[\omega(t,y,z) - u_t(t,y,z)h_y(t,y,z) + u_y(t,y,z)h_t(t,y,z)\right]\right]$$

$$h(t,y,z) = T^{-1}[p(s)q^{-\alpha}h(t,y,0)] + T^{-1}\left[q^{-\alpha}T\left[h(t,y,z) - u_t(t,y,z)\omega_y(t,y,z) + u_y(t,y,z)\omega_t(t,y,z)\right]\right]$$

As before the first few terms of u(t, y, z), ω(t, y, z) and h(t, y, z) in this case are:

$$\begin{cases} u_0(t,y,z) = e^{t+y} \\ \omega_0(t,y,z) = e^{t-y} \\ h_0(t,y,z) = e^{-t+y} \end{cases}$$

$$\begin{cases} u_1(t,y,z) = -\dfrac{e^{t+y}z^{\eta}}{\Gamma(\eta+1)} - \dfrac{e^{t-y}e^{-t+y}z^{\eta}}{\Gamma(\eta+1)} - \dfrac{e^{t-y}e^{-t+y}z^{\eta}}{\Gamma(\eta+1)} = \dfrac{e^{t+y}z^{\eta}}{\Gamma(\eta+1)} \\ \omega_1(t,y,z) = \dfrac{e^{t-y}z^{\mu}}{\Gamma(\mu+1)} - \dfrac{e^{t+y}e^{-t+y}z^{\mu}}{\Gamma(\mu+1)} - \dfrac{e^{t+y}e^{-t+y}z^{\mu}}{\Gamma(\mu+1)} = \dfrac{e^{t-y}z^{\mu}}{\Gamma(\mu+1)} \\ u_1(t,y,z) = \dfrac{e^{-t+y}z^{\gamma}}{\Gamma(\gamma+1)} - \dfrac{e^{t+y}e^{t-y}z^{\gamma}}{\Gamma(\gamma+1)} - \dfrac{e^{t+y}e^{t-y}z^{\gamma}}{\Gamma(\gamma+1)} = \dfrac{e^{-t+y}z^{\gamma}}{\Gamma(\gamma+1)} \end{cases}$$

$$\begin{aligned} u_2(t,y,z) = {} & \frac{e^{t+y}z^{2\eta}}{\Gamma(2\eta+1)} - \frac{e^{t-y}e^{-t+y}z^{\alpha+\gamma}}{\Gamma(\eta+\gamma+1)} - \frac{e^{t-y}e^{-t+y}z^{\eta+\mu}}{\Gamma(\eta+\mu+1)} \\ & - \frac{\Gamma(\gamma+\mu+1)e^{t-y}e^{-t+y}z^{\eta+\mu+\gamma}}{\Gamma(\mu+1)\Gamma(\gamma+1)\Gamma(\eta+\mu+\gamma+1)} + \frac{e^{t-y}e^{-t+y}z^{\eta+\gamma}}{\Gamma(\eta+\gamma+1)} \\ & + \frac{e^{t-y}e^{-t+y}z^{\eta+\gamma}}{\Gamma(\eta+\mu+1)} + \frac{\Gamma(\gamma+\mu+1)e^{t-y}e^{-t+y}z^{\eta+\mu+\gamma}}{\Gamma(\mu+1)\Gamma(\gamma+1)\Gamma(\eta+\mu+\gamma+1)} \\ = {} & \frac{e^{t+y}z^{2\eta}}{\Gamma(2\eta+1)} \end{aligned}$$

$$\begin{aligned} \omega_2(t,y,z) = {} & \frac{e^{t-y}z^{2\mu}}{\Gamma(2\eta+1)} - \frac{e^{t+y}e^{-t+y}z^{\gamma+\mu}}{\Gamma(\gamma+\mu+1)} - \frac{e^{t+y}e^{-t+y}z^{\eta+\mu}}{\Gamma(\eta+\mu+1)} \\ & + \frac{\Gamma(\gamma+\eta+1)e^{t+y}e^{-t+y}z^{\eta+\mu+\gamma}}{\Gamma(\mu+1)\Gamma(\gamma+1)\Gamma(\eta+\mu+\gamma+1)} + \frac{e^{t+y}e^{-t+y}z^{\gamma+\mu}}{\Gamma(\gamma+\mu+1)} \\ & - \frac{e^{t+y}e^{-t+y}z^{\eta+\mu}}{\Gamma(\eta+\mu+1)} + \frac{\Gamma(\gamma+\eta+1)e^{t+y}e^{-t+y}z^{\eta+\mu+\gamma}}{\Gamma(\eta+1)\Gamma(\gamma+1)\Gamma(\eta+\mu+\gamma+1)} \\ = {} & \frac{e^{t-y}z^{2\mu}}{\Gamma(2\mu+1)} \end{aligned}$$

$$\begin{aligned} h_2(t,y,z) = {} & \frac{e^{-t+y}z^{2\gamma}}{\Gamma(2\gamma+1)} - \frac{e^{t+y}e^{t-y}z^{\gamma+\mu}}{\Gamma(\gamma+\mu+1)} - \frac{e^{t+y}e^{t-y}z^{\eta+\gamma}}{\Gamma(\eta+\gamma+1)} \\ & - \frac{\Gamma(\eta+\mu+1)e^{t+y}e^{t-y}z^{\eta+\mu+\gamma}}{\Gamma(\eta+1)\Gamma(\mu+1)\Gamma(\eta+\mu+\gamma+1)} - \frac{e^{t+y}e^{t-y}z^{\gamma+\mu}}{\Gamma(\gamma+\mu+1)} \\ & + \frac{e^{t+y}e^{t-y}z^{\eta+\gamma}}{\Gamma(\eta+\gamma+1)} + \frac{\Gamma(\eta+\mu+1)e^{t+y}e^{t-y}z^{\eta+\mu+\gamma}}{\Gamma(\eta+1)\Gamma(\mu+1)\Gamma(\eta+\mu+\gamma+1)} \\ = {} & \frac{e^{-t+y}z^{2\gamma}}{\Gamma(2\gamma+1)} \end{aligned}$$

Therefore, the series solutions can be written in this form

$$u(t,y,z) = e^{t+y} - \frac{e^{t+y}z^{\eta}}{\Gamma(\eta+1)} + \frac{e^{t+y}z^{2\eta}}{\Gamma(2\eta+1)} + \cdots = e^{t+y}\left(1 + \sum_{i=1}^{\infty}\frac{(-z^{\eta})^i}{\Gamma(i\eta+1)}\right) = e^{t+y}E_t(-z)^{\eta}$$

$$\omega(t,y,z)=e^{t-y}-\frac{e^{t-y}z^{\mu}}{\Gamma(\mu+1)}+\frac{e^{t-y}z^{2\mu}}{\Gamma(2\mu+1)}+\cdots=e^{t-y}\left(1+\sum_{i=1}^{\infty}\frac{(-z^{\mu})^{i}}{\Gamma(i\mu+1)}\right)=e^{t+y}E_{\mu}(-z)^{\mu}$$

$$h(t,y,z)=e^{y-t}-\frac{e^{y-t}z^{\gamma}}{\Gamma(\gamma+1)}+\frac{e^{y-t}z^{2\gamma}}{\Gamma(2\gamma+1)}+\cdots=e^{-t+y}\left(1+\sum_{i=1}^{\infty}\frac{(-z^{\gamma})^{i}}{\Gamma(i\gamma+1)}\right)=e^{y-t}E_{\gamma}(-z)^{\gamma}$$

we put $\eta=\mu=\gamma=1$ we get.

$$u(t,y)=e^{t+y}\left(1-z+\frac{z^2}{2!}-\frac{z^3}{3!}+\cdots\right)=e^{t+y-z}$$

$$\omega(t,y)=e^{t-y}\left(1+\frac{z^2}{2!}+\frac{z^3}{3!}+\cdots\right)=e^{t-y\mp z}$$

$$h(t,y)=e^{-t+y}\left(1+\frac{z^2}{2!}-\frac{z^3}{3!}+\cdots\right)=e^{-t+y-z}$$

5. Conclusion:

In this paper, we dealt with a new method, which is the iterative T-transformation method. It is considered more general in relation to the previous transformations, and we applied it in our work to derive accurate and approximate analytical solutions for fractional order partial differential equations. We have shown that this method can reduce the amount of computational work compared to the traditional methods. Also, this method has a clear advantage over the methods of decomposition and symmetric analysis in solving nonlinear problems. Since lTTM does not need to calculate polynomials. Finally, we conclude that this method can be considered as a good improvement in numerical techniques. Two examples are presented with their results, for the specific cases.

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