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FINDING THE BEST SOLUTION TO SOLVE THE TRANSPORTATION PROBLEM WITHIN THE BUDGET CONSTRAINT (Case Study)

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Abstract:

The problem of transport is one of the most important methods that contribute to the appropriate decision-making in the transport of goods or materials from their sources of manufacture or production to their requesting entities at the lowest possible cost. In this research we will clarify the mathematical transport model of the transport problem of different cost vehicles and using a modern algorithm that contributes to the optimal solution. This model has proven its efficiency in reducing the total transportation costs of the product, and the results show that the final cost of the stores is as high as (19) thousand dinars compared to the budget set for the cost of transportation which amounted to (50) thousand dinars and it is clear from the application of this model that it has achieved profits in value (31) thousand dinars, so the modern algorithm is one of the methods that helps decision makers to make the right decision in productive and economic establishments being easy to apply and more efficient in application.

Key terms of research: transfer model for different cost vehicles, budgeted, optimal solution.

First Search

1-1 Introduction:

The transport problem is one of the most important problems concerned with transporting goods and raw materials from their storage to their requesting destinations, which requires the construction of a mathematical model that reduces the cost of transportation using written programming or one of the methods of optimal transportation solution. This model is known as the general model in which the vehicle is of one type and the quantities offered and required.

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With the evolution of methods of solving the transport problem, the problem requires a planning study of transport from its sources to its requesting party, since the mode of transport with a different transport cost, hence the need to build a mathematical model in which the means of transport and its cost are different.

2-1: Research Problem

From the previous studies and research, it is clear that the transport problems dealt with the transport of goods from the source of the processed at a specific cost, but the actual reality does not exist. Therefore, a realistic and efficient model has to be found that contributes to solving the problem. In this research, we will address the problem of the transport of food products to the Ministry of Trade for flour, given the urgent need for the substance and the varying quantity of demand for it which makes it difficult to determine the actual final cost of transporting the substance. This algorithm has therefore been used and has proven to be efficient in determining the final cost and within the transport budget.

3-1: Research objective:

The aim of the research is to develop the mathematical model of the public transport problem by adding a budget entry for the purpose of solving the transport problem when there are two transport vehicles to optimize the solution of the lowest possible transportation cost.

The research includes three themes:

First ": includes the introduction, the problem of research and its purpose from this study

Second ": The theoretical theme that clarifies the basic concepts of the transport model and the problem of transport and the formulation of its own mathematical model

Third ": The practical theme of data documentation for the application of the modern algorithm to solve the problem of transport when there are two costs with results analysis.

IV ": conclusions and recommendations

Included the most important findings and recommendations of the research and analyzed them in the light of the findings.

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2- Theoretical aspect

1-2: linear programming concepts

It is an analysis tool to find the optimal use of the capabilities and resources available to the enterprise to come up with the decision facing management in the enterprises, Programming is one aspect of this method, which is the possibility of using this tool. It does not find the different programs in order to invest all the resources available in that institution because there are limitations on the available capabilities and by choosing the best programs that achieve the goal that the establishment seeks, either with regard to the other side the linear degree. Linearity means all linear programming equations (i.e. limitations) that are first class and that are represented in a straight line.

The most important components of the linear programming model of three are:

1. Objective function is of the kind that reduces the cost or maximizes profits.
2. Decision variables to be determined.
3. Structural constraints to be achieved in solving the problem (1).

2-2: Transport Model

The concept of the transport model is based on the economic transfer of the units produced in the installations in the sense that there is one substance transported from one or more production sources to several requesting or consuming entities to achieve the lowest mechanized costs. The most important conditions to be met in this model are:

1. Harmonization of all transferred units
2. Balancing supply quantities with required quantities
3. The cost of transferring the unit from the source to the requisitioner is fixed
4. Transportation cost has no direct link to the number of units transferred (2)

3-2: Mathematical version of the transport model

The transport form consists of m: sources of processing and n: applicants in addition to the following (6):

- a_i: Units displayed in sources i = 1,2,..., m
- b_j: Number of units required for j = 1,2,..., n
- c_{ij} : Cost of moving one unit from source i to location j
- x_{ij} : represents the quantity transferred from source i to location j

$$\text{Minimize } (z) = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

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Subject to

= a_i , i=1,2,....,m $\sum_{j=1}^{n} x_{ij}$

(1)

= b_i , j=1,2,....,n (2)$\sum_{i=1}^{m} x_{ij}$

$x_{ij} \geq 0$, i = 1,2, ... , m , j = 1,2, ... , n

Methods used to solve transport problem
Common ways to reach the first basic solution are:

1. North West Corner Method
2. Low-cost method
3. Vogel's approximate method

In this study we will use the northwest corner method as it depends on the location of the cells and not on the cost and the following are the most important steps to apply this method (5)

Northwest Corner Method

This method is used in the event of emergency and disasters and does not take into account the cost of transport from the source to its consumption center, but rather the cell's location for the northwest corner of the transport schedule and heads southward to reach the south-eastern corner and therefore does not depend on the cost as a determining factor in the allocation method (4)

As for the process of improving the acceptable primary basic solution extracted from the Northwest Corner method after confirming that the core cells are achieving the basic requirement (m + n-1), Modified Distribution Method (MODJ) is used after ascertaining the possibility of developing the solution to optimize the solution (2)

4-2: Two- vehicle cost varying transport problem

The classic transport problem in which a particular commodity is transferred from the sources to all its requesting entities in order to reduce the final cost while identifying the basic variables, i.e. the quantities that are transferred from the sources to their consumers, but the economic enterprises seek to increase their profits using all available means and developed methods.

Through this, a modern method has been developed to solve the problem of transportation with two types of vehicles intended for transportation and two different costs, in addition to a restriction, which is a budget determined by the enterprise for the total cost of transportation.

Suppose there are two types of vehicles and code them as $V_{1\,,}$, $V_{2\,,}$to transfer from each source to each requesting entity$C_{1\,,}$, C_2 represent the absorptive capacity of the two vehicles $V_{1\,,}$, $V_{2\,,}$provided they are $C_2 \; > C_{1\,,}$

R_{ij}: represents the transport cost of the first and second vehicle, since

R_{ij}^1represents the cost of one unit from source i to request j for the first vehicle

R_{ij}^2 represents the per unit cost from source i to request j for vehicle II

The transport problem can be explained by the default table below.

	D_1	D_2	D_3	presentation
O_1	$R_{11}^1.\ R_{11}^2$	$R_{12}^1.\ R_{12}^2$	$R_{13}^1.\ R_{13}^2$	a_1
O_2	$R_{21}^1.\ R_{21}^2$	$R_{22}^1.\ R_{22}^2$	$R_{23}^1.\ R_{23}^2$	a_2
O_3	$R_{31}^1.\ R_{31}^2$	$R_{32}^1.\ R_{32}^2$	$R_{33}^1.\ R_{33}^2$	a_3
request	b_1	b_2	b_3	

Table No. (1) shows the Two- vehicle cost varying transport problem

This model uses the Northwest Corner method to extract the initial solution to the problem because it does not depend on the cost of transport but the location of the cells and then uses the multiplier method to improve the solution after ensuring that the number of fundamental variables achieves the requirement (m + n-1).

5-2: The algorithm method for solving the two-vehicle transport problem (3)

5-2-1: In order to apply the modern algorithm to solve the problem, the cost C_{ij} must be determined according to the following:

First case max $a_i \;\le\; C_{2.i}$

5-2-2: Type I algorithm (TP1)

Step 1 - In this case the cost is not determined to be dependent on the quantity of transport, as the Northwest Corner method was used because the method depends on the location and not the cost as mentioned earlier and the cells are allocated in the initial basic solution.

Step 2 - After the process of determining the quantities transferred x_{ij} in the north-west corner method, and to determine the core cells we identify the C_{ij})that represent the cost of moving the unit from the $O_i$source of the order $D_j$as follows:

$$C_{ij} = \begin{cases} \dfrac{R_{ij}^1}{x_{ij}} & if \quad x_{ij} \leq C_1 \\ \dfrac{R_{ij}^2}{x_{ij}} & if \quad C_1 < x_{ij} \leq C_2 \\ 0 & if \quad x_{ij} = 0 \end{cases} \quad \ldots\ldots(3)$$

Step 3: After the previous step, we identify the non-core cells after the process of customizing the cells with the lowest presentation and request of the core cells where we determine the cost of the non-core cells C_{ij} which represents the cost of transferring the unit from the source to the requisitioner according to the equation below:

$$C_{ij} = \begin{cases} \dfrac{R_{ij}^1}{x_{ij}} & if \quad x_{ij} \leq C_1 \\ \dfrac{R_{ij}^2}{x_{ij}} & if \quad C_1 < x_{ij} \leq C_2 \\ 0 & if \quad x_{ij} = 0 \end{cases} \quad \ldots\ldots(4)$$

In this case we turn the two- vehicle cost varying transport problem into the traditional transport problem but C_{ij} is not fixed, it is possible to change when the allocation does not give optimal value during the ideal test of the problem.

Step 4: By testing optimal to solve the problem, some core cells are transformed into non-core cells. Conversely, some non-core cells are transformed into core cells depending on the allocation of the core cells, where we first modify C_{ij} by step (2) and non-core cells we correct the cost in step (3).

Step 5: We repeat the steps from 2 to 4 until the optimal solution to the transport problem is reached.

5-2-3: Type II algorithm (TP2)

Step 1: Determine the cost using the Northwest Corner method because the method depends on the location and not the cost, and then we allocate the cells in the first basic solution.

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Step 2 - After determining the quantities transferred x_{ij} in the above manner, after determining the core cells we identify the C_{ij})which represents the cost of moving the unit from the $O_i$source of the request $D_j$as follows:

$$C_{ij} = \begin{cases} \dfrac{R_{ij}^1}{x_{ij}} & if \quad x_{ij} \leq C_1 \\ \dfrac{R_{ij}^2}{x_{ij}} & if \quad C_1 < x_{ij} \leq C_2 \\ \dfrac{R_{ij}^1 + R_{ij}^2}{x_{ij}} & if \quad C_2 < x_{ij} \leq C_1 + C_2 \\ 0 & if \quad x_{ij} = 0 \end{cases} \quad \ldots\ldots..(5)$$

Step 3: After the process of customizing the cells with the lowest presentation and request, we identify the non-core cells of the core cells where we determine the cost of the non-core cells C_{ij} ,which defines the cost of transferring the unit from each source to each destination according to the equation below

$$C_{ij} = \begin{cases} \dfrac{R_{ij}^1}{x_{ij}} & if \quad x_{ij} \leq C_1 \\ \dfrac{R_{ij}^2}{x_{ij}} & if \quad C_1 < x_{ij} \leq C_2 \\ \dfrac{R_{ij}^1 + R_{ij}^2}{x_{ij}} & if \quad C_2 < x_{ij} \leq C_1 + C_2 \\ 0 & if \quad x_{ij} = 0 \end{cases} \quad \ldots\ldots..(6)$$

In this algorithm we turn two- vehicle cost varying transport problem into the normal transportation problem but C_{ij} is uneven, then we change when the allocation does not give optimal value when testing the solution improvement of the problem.

Step 4: When applying the optimal test to solve the problem, there are some core cells that turn into non-core cells, as well as some non-core cells that turn into core cells based on the allocation of core cells, where we modify C_{ij} according to step (2) and non-core cells we adjust the transport cost as in step (3).

Step 5: Repeat step from (2) to step (4) until we reach the ideal solution in solving the transport problem.

5-2-4: Type III algorithm (TP3)

Step 1: Extract the primary basic solution in the way of the northwest corner as explained in the first and second type.

Step 2: After the customization process, we determine the cost of the core cells from each source to each consumer

$$C_{ij} = \begin{cases} \frac{p_{ij}^{1} \quad R_{ij}^{1} + p_{ij}^{2} \quad R_{ij}^{2}}{x_{ij}} & if \quad x_{ij} \neq 0 \\ 0 & if \quad x_{ij} = 0 \end{cases} \quad \ldots\ldots\ldots (7)$$

Where p_{ij}^{1} , p_{ij}^{2} is the right solution, so:

Min $p_{ij}^{1} \quad R_{ij}^{1} + p_{ij}^{2} \quad R_{ij}^{2}$

S.t. $x_{ij} \leq p_{ij}^{1} \quad C_1 + p_{ij}^{2} \quad C_2$

Step 3: For variables for non-core cells and within the possible allocation of the lowest allocation for each row and column, the allocation can be made for x_{ij} then the cost of non-core cells is any transport cost for each unit of the O_i source of the order D_j as follows:

$$C_{ij} = \begin{cases} \frac{p_{ij}^{1} \quad R_{ij}^{1} + p_{ij}^{2} \quad R_{ij}^{2}}{x_{ij}} & if \quad x_{ij} \neq 0 \\ 0 & if \quad x_{ij} = 0 \end{cases} \quad \ldots\ldots\ldots (8)$$

Where p_{ij}^{1} , p_{ij}^{2} is the right solution, so:

I=1,2,……,m

J=1,2,……,n

Min $p_{ij}^{1} \quad R_{ij}^{1} + p_{ij}^{2} \quad R_{ij}^{2}$

In case $x_{ij} \leq p_{ij}^{1} \quad C_1 + p_{ij}^{2} \quad C_2$

In this case, the cost varying transport problem turns into a classic transportation problem, considering that the cost variable as well as the customization process does not give real value during the solution testing process to reach the optimal solution.

Step 4: During the process of improving the solution, there are some non-core cells that turn into core cells, and vice versa, as evidenced by Step 2 and Step 3.

Step 5: Repeat steps from (2) to (4) until optimal vinegar is obtained.

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5-2-5: Mathematical model for the problem of bi-cost transport

The bi-level mathematical formula for two different-cost vehicles for transport problem is as follows:

Min $z=\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}\, x_{ij}$ ……(9)

$$C_{ij} = \begin{cases} \dfrac{p_{ij}^1 \quad R_{ij}^1 + p_{ij}^2 \quad R_{ij}^2}{x_{ij}} & if \quad x_{ij} \neq 0 \\ 0 & if \quad x_{ij} = 0 \end{cases}$$

The C_{ij} is determined by the right solutions and sports programming through

$p_{ij}^1 \ , p_{ij}^2$

I=1,2,……,m

J=1,2,……,n

And Min $p_{ij}^1 \quad R_{ij}^1 + p_{ij}^2 \quad R_{ij}^2$

In case $x_{ij} \leq p_{ij}^1 \quad C_1 + p_{ij}^2 \quad C_2$

$$\sum_{i=1}^{m} x_{ij} = a_i \qquad , i = 1,2, \dots m$$

$$\sum_{j=1}^{n} x_{ij} = b_j \qquad , j=1,2, \dots . n$$

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

$$x_{ij} \geq 0 \ \forall i , \forall j$$

Practical aspect

3-1: Preface

The transport problem in many studies and research on the problem is the transport of goods from sources to the recipient and as a result of the evolution of modern transport methods imposed the need to apply modern algorithms on them that give results to decision makers in the decision-making process. In this research we will apply what has been exposed on the theoretical side using a modern technique to solve the problem of the transfer of flour material from its stores to the requesting markets of the product by two different vehicles for a cost and within a specific budget for transportation by the enterprise beforehand.

The data related to the storage capacity of the three stores and the quantities required in the sales centers have been documented. The cost is based on distance, quantity, transportation price and use of two vehicles with a certain cost. All information is scheduled in Table 2 below.

	Center 1	Center 2	Center 3	Center 4	Presentation
A	6,9	8,13	5,10	3,6	200
B	10,15	4,8	4,6	2,8	150
C	12,15	7,14	5,9	3,8	100
Request	150	95	100	80	

Table No. (2) shows the problem of transfer of flour

Knowing that the vehicle's capacity is as follows:

$$(c_1 = 100 , \quad c_2 = 200)$$

With a budget determined by the flour plant, the cost of transportation is (50) thousand dinars.

Step 1: After balancing the transport schedule, the northwest corner method is used to extract the accepted primary basic solution to determine the decision variables of the transport problem as in the table below:

	Center 1	Center 2	Center 3	Center 4	Fake center	Presentation
A	150 6,9	50 8,13	5,10	3,6	0	200
B	10,15	4,8 45	4,6 100	2,8 5	0	150
C	12,15	7,14	5,9	75 3,8	25 0	100
Request	150	95	100	80	25	

Table No. (3) shows the application of the northwest corner method to the problem of transfer of flour

Step 2: Determine the special cost of transportation according to the modern algorithm by equation number (3). (4) Agencies:

From the equation (3) to determine the core cells in the transport matrix:

$$(c_{11} = 3/50 \quad , c_{12} = 4/25, \quad c_{22} = 4/45 , \; c_{23} = 1/25 \quad , c_{24} = \frac{2}{5} , c_{34} = \frac{1}{25} , \; c_{35} = 0)$$

From the equation (4) we identify the non-fundamental variables of the transport schedule as below:

$$(c_{13} = 1/10 \quad , c_{14} = 3/50, \quad c_{15} = 0 , \; c_{21} = 2/9 \quad , c_{25} = 0 , \; c_{31} = \frac{4}{25} , c_{32} = \frac{7}{75} , c_{33} = \frac{1}{15})$$

Step 3: After the transporting cost of the core and non-core cells has been determined by the equation (3.4), we test the optimal solution after the requirement to improve the acceptable solution we got (m + n-1)

By baseline cell multipliers by c_ equation $c_{ij} = u_i + v_j$) and for non-baseline cells $d_{ij} = c_{ij} - u_i - v_j$) as shown in the table below

	Center 1	Center 2	Center 3	Center 4	Fake Center	
A	150 3 /50	4 /25 50	1/10	3/50	0	u_1
B	2 / 9	45 4/45	1/25 100	2/5 5	0	u_2
C	4/25	7/75	1/15	1/25 75	0 25	u_3
Request	v_1	v_2	v_3	v_4	v_5	

Table No. (4) shows the application of multiplier method to transport problem

Step 4: After repeating the previous steps in the method of multiplying factors so that they can be$d_{ij} \geq 0$ then we find the optimal solution to the transport problem as the table below shows:

	Center 1	Center 2	Center 3	Center 4	Fake center	
A	150 3 /50	/25 4	1/10	3/50 50	0	200
B	2 / 9	25 4/45	100 1/25	2/5	0 25	150
C	4/25	7/7570	1/15	1/25 30	0	100
Request	150	95	100	80	25	

Table No. (5) shows the process of customizing cells after determining the optimal solution

So specific cells can be identified as follows and with resolution variables as follows:

$$x_{11} = 150$$
$$x_{14} = 50$$
$$x_{22} = 25$$
$$x_{23} = 100$$
$$x_{32} = 70$$
$$x_{34} = 30$$

By allocating cells, the lowest possible cost of transportation can be extracted: (3 + 3 + 4 + 1 + 7 + 1) = 19

So the total cost of transporting flour from the plant to the centers was 19, that is, within the budget allocated by the plant for transporting it.

two- vehicle cost varying transport problem Two–vehicle CVTP , V_1 , V_2	Transport problem for the first and second vehicle	
	Second Vehicle V_2	V_1 First Vehicle
Z = 19 $x_{11} = 150$ $x_{14} = 50$ $x_{22} = 25$ $x_{23} = 100$ $x_{32} = 70$ $x_{34} = 30$	Z=46 $x_{11} = 150$ $x_{14} = 50$ $x_{22} = 95$ $x_{23} = 55$ $x_{33} = 45$ $x_{34} = 30$ $x_{35} = 25$	Z=25 $x_{11} = 150$ $x_{13} = 45$ $x_{14} = 5$ $x_{22} = 95$ $x_{23} = 55$ $x_{34} = 75$ $x_{35} = 25$

Table No. (6) shows the comparison of results with the determination of decision variables

Conclusions:

In this research, the transport problem of two vehicles of varying cost was studied after the application of the Northwest Corner method and the testing optimal solution, after comparing the results and determining the decision variables through the allocation of cells. We conclude that this modern algorithm is highly efficient and easy to apply to decision makers in order to optimize the solution in productive and economic installations. Its efficiency has also proved to be more realistic in determining the final transport cost which is within the facility's budget as compared to the final cost determined for each vehicle without exceeding the budget prescribed by the enterprise for transporting the material.

Arab Sources:

Bakhit, Abdul-Jabbar Khader, "Solving the problem of three-dimensional transport using linear programming", 2017, Journal of Economic and Administrative Sciences, Issue 103, vol. 24, p. 395- 410.

2. Janabi, Hussein Mahmoud ", the latest in operational research", (2010), Oman Dar al-Hamid 2010.

Foreign Sources:

3- Panda .A.DAS.C.B.," 2-Vehicle cost varying transportation problem ", (2014), journal of uncertain systems ,vol 8, no.1, pp.44-57.3

4- Seethalakshmy .A, Srinivasan.N , " Anew Technique to Obtain Initial Basic Feasible Solution for the Transportation Problem " ,(2016), international Journal for Research in Applied Science & Engineering Technology (IJRASET),vol.4,ISSUE X ,ISSN:2321-9653,P.P: 542-547.

5- Seethalakshmy .A, Srinivasan.N,"An Improved Algorithm to Obtain Initial Basic Feasible Solution for the Transportation Problem", (2015),International Journal of Science and Research (IJSR) ,index Copernicus value (2015):78-96 impact factor (2015) :6.391,ISSN:2319-7064, p.p. 1225-1229.

6- Yamini .D.l,Srinivasan.N,(2016),"Anew Method to Find Initial Basic Feasible Solution to Transportation Problem", International Journal of Engineering and Management Research ,vol(6), ISSUE(5), P.P.302-305.